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Can we really ignore time in Simple Plant Location Problems?

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In simple plant location problems (SPLP), the time dimension is not explicitly considered, either because there are not significant costs for relocating facilities, or because the assignment costs are not expected to change significantly as time goes by. Nevertheless, location problems are strategic decisions by nature. In this paper, we will show how the explicit consideration of a planning horizon, as well as the explicit definition of time dependent assumptions, is essential in the definition and application of SPLPs because they can influence significantly the optimal decision.

Keywords: location problems; planning horizon; discount rate; equivalent annual cost
AMS Subject Classification: 90C10; 90B80; 90C27.

1. Introduction

Simple plant location problems (SPLP) are, possibly, one of the most studied location problems of all time. Considering a set of locations where facilities can be opened (at most one facility at each location), and considering a set of clients, the problem consists of finding the best set of locations where facilities will be opened, guaranteeing that each client is assigned to exactly one opened facility and minimizing total costs: fixed costs associated with opening each facility and assignment costs related to the assignment of each client to an open facility.

SPLPs can be applied if a set of assumptions are fulfilled, namely:

a) There are no capacity constraints associated with the facilities. This means that it would be possible to have all clients assigned to one and only one opened facility.
b) Assignment costs are not expected to change significantly during the lifetime of the facilities, or if they do change, the change will be of similar order of magnitude for all costs.
c) Decisions regarding the location of facilities will be taken at the present moment, and it is not necessary to plan now the opening or closing of facilities in the future.

If assumption a) is not fulfilled, then we should consider capacitated location problems, where each facility has an upper limit to the total amount of demand it can serve, or an upper limit to the total number of clients that can be assigned to it. If assumptions b) and c) are not fulfilled, then dynamic location problems should be used, where time is explicitly considered.

Although SPLPs do not consider *time* explicitly, most of the times these problems consider strategic decisions, difficult to revert and with consequences that spread over long time periods. In fact, when we define fixed and assignment costs, special care should be taken regarding the way these costs are calculated, because they should reflect what is expected to happen during the lifetime of the facilities. The fixed cost associated with opening a given facility should represent not only the fixed opening cost, but also the maintenance and operational costs during the facility's operational period and possibly costs incurred when the facility is closed and its salvage value. Assignment costs should reflect the costs associated with assigning clients to facilities during the whole lifetime of the facilities. So, even when time is not explicitly considered, an implicit assumption regarding the definition of a planning horizon has to be considered. In fact, this is true not only for simple plant location problems but for all static location problems, and it is of particular importance when we are dealing with the application of these mathematical models to real world problems. If we are dealing with facilities that show different patterns of fixed and variable costs along the planning horizon, then it is necessary to address the problem of how the overall cost will be calculated, and the explicit assumption of a discount rate, for instance, is essential. We should also consider whether all facilities have the same lifespan. If not, a simple comparison of the cost flows associated with the facilities is improper, and we should resort to concepts like the equivalent annual cost. Regarding the location literature, we can see that, most of the times, information about how the objective function parameters should be calculated are absent, and sometimes values of completely different nature and order of magnitude are simply summed up together in the objective function. There are some few good examples. In [1], for instance, there is an explicit consideration of annual equivalent costs associated with the facilities, although there is no explanation about the assumptions made in this calculation. In [5], the authors explain how they have used a static location problem considering the long-run effects of the decisions and minimizing the present value of total costs. In [3], the authors study the problem of locating slaughterhouses under economies of scale, and carefully explain how costs are incorporated into the model. A location-routing problem applied to the location of incinerators for the disposal of solid animal waste is studied in [4], and all costs were calculated considering the period the incinerator will be in service, but without further details. In [6, 7], the authors describe the problem of locating and deciding the capacity of plants for bottling propane in south India, and consider the fixed location costs as being calculated using cash-flow patterns associated with each given plant and size when the plant is operating at full capacity.

In this paper we intend to show that time should be explicitly considered in SPLPs, and that there are consequences of not taking time into account when applying these models: we can end up with a suboptimal solution.

In the next section we define the SPLP, and describe two different ways of considering time in SPLP: using future cash flows, discounted at a given discount rate, or using the equivalent cost concept [2]. In section 3 we show some computational results. Section 4 states the main conclusions.

2. Simple Plant Location Model

The simple plant location problem can be defined as follows:

$$Min \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{i \in I} f_i y_i \qquad (1)$$

Subject to: (2)

$$\sum_{i \in I} x_{ij} = 1, \forall j \qquad (3)$$

$$x_{ij} \leq y_i, \forall i, j \qquad (4)$$

$$y_i \in \{0,1\}, x_{ij} \in \{0,1\} \qquad (5)$$

Where:

$I =$ set of possible locations for facilities

$J =$ set of clients

$$y_i = \begin{cases} 1, & \text{if a facility is open at location } i \\ 0, & \text{otherwise} \end{cases}, \forall i \in I$$

$$x_{ij} = \begin{cases} 1, & \text{if client } j \text{ is assigned to facility open at location } i \\ 0, & \text{otherwise} \end{cases}, \forall i \in I, j \in J$$

$c_{ij} =$ cost of assigning client j to facility located at $i, \forall i \in I, j \in J$

$f_i =$ fixed cost of opening a facility at location $i, \forall i \in I$

The objective function minimizes total cost (assignment costs plus fixed costs associated with the opening of the facilities), constraints (2) guarantee that each client will be assigned to exactly one facility, constraints (3) guarantee that clients will only be assigned to opened facilities. The location variables y_i are binary. The assignment variables x_{ij} can be considered as binary variables, or $x_{ij} \in [0,1]$. As we are not considering capacity constraints, in the optimal solution each client will always be assigned to the facility that has the minimum assignment cost, so x_{ij} will always be 0 or 1, even if that is not explicitly considered in the model.

Our attention will be focused on the objective function, mainly considering how should the values of c_{ij} and f_i be calculated.

2.1. Fixed costs

Let us first consider the fixed location costs f_i. What do these costs represent? When we think about opening a new facility, several different situations can be considered: we may have to build the facility, and even build some infrastructures; we may already own the building and opening a facility will require the acquisition of machinery, for instance; we may be renting a warehouse; and so on. Different situations will have different costs associated, but what is important to notice is that, in general, these fixed costs will not be incurred entirely at the present time (at the time when the decision is being made). In general, these fixed costs will arise in different time periods. We can imagine that in the first years the cost flows will be greater, corresponding to the setting up of the facility. Once the facility is operating at its full potential, then there will be fixed maintenance and operating costs that have to be considered. At the end of the facility's lifetime, it is still possible to consider a negative cost, or a benefit, usually denominated salvage value, that can be interpreted as the remaining value of the asset. When talking about facilities, salvage values can be significant due to the usual low depreciation rate associated.

When we are considering different possible locations for facilities, we may be facing a situation of comparing locations with completely different cost flow patterns, so care must be taken when defining f_i values. One way of solving this issue is to resort to the concept of present value. Present value allows us to discount future costs so that they are all in a common metric and can then be comparable. Defining f_i as the present value of all the fixed costs associated with opening one facility at location *i* has implicit the assumption of a given planning horizon and a given discount rate. The planning horizon can be defined as the number of time periods (let us consider years, for ease of the exposition) that the facility is expected to be in operation. The discount rate can be seen as representing the time value of money (we prefer to receive the same amount of money today than to wait, so if we are willing to wait we should be compensated by receiving more) and also a risk premium (we want to be compensated by the risk we are taking with the investment). In the location problem considered, we are dealing with a deterministic problem, with no uncertainty associated, so we can consider the discount rate as representing the time value of money alone. This means that we could consider using a risk-free rate as our discount rate.

Consider the following example: There are two possible locations for locating warehouses, location A and location B. Location A has already a warehouse that we can rent by 20000€ a year. Location B will force us to build the warehouse from scratch, with an initial cost of 165000€ in the first year, and then maintenance costs of 3000€ per year. We are thinking about using these facilities during 10 years. How should f_1 and f_2 be calculated?

We should consider all the costs associated with each potential location in each year of the planning horizon, as shown in Table 1.

Table 1 – Cost flows associated with two facilities

Year	Location A	Location B
1	20 000€	165 000€
2	20 000€	3 000€
3	20 000€	3 000€
4	20 000€	3 000€
5	20 000€	3 000€
6	20 000€	3 000€
7	20 000€	3 000€
8	20 000€	3 000€
9	20 000€	3 000€
10	20 000€	3 000€
PV (f_i)	**179 652€**	**185 771€**

The present value (PV) of a flow of costs $C_t, t = 1,...,T$, considering a discount rate r can be calculated as follows[1] [2]:

[1] In this case we are considering that costs are incurred at the end of the corresponding time period. We could also consider that the costs would be incurred at the beginning of the time period, by considering $t = 0,...,T-1$: $PV = \sum_{t=0}^{T-1} \frac{C_t}{(1+r)^t}$. The present values would be slightly changed to 183 245€ and 188 487€.

$$PV = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} \qquad (6)$$

With r equal to 2%, opening the facility at A will have a fixed cost of 179 652€ and opening a facility at B will have a fixed cost of 185 771€.

But imagine now that after 10 years, we would be able sell the warehouse located at B, so that we would have a benefit at the end of the warehouse's lifetime. Imagine that the benefit could be estimated in 75 000€. This value should also be taken into account in the calculation of the present value f_2, that would now be decreased to 124 245€.

In the previous example we considered that we will be able to use the facilities during the whole planning horizon. But what if we are dealing with facilities that have different lifespans? Facilities with different time frames cannot be directly compared, because if we calculate fixed costs as shown in (6) we will be having some facilities accumulating more costs than others. Imagine, for instance, that we want to install new plants, and in each potential location we can consider building a facility that is expected to last for 5 years and/or building a facility that is expected to be in operation during 10 years[2]. Calculating the present values associated with each one of the options for a given location *i*, imagine we end up with f_i' = 200 000€ and f_i'' = 300 000€, where f_i' refers to the present value associated with the 5-years option, and f_i'' refers to the present value associated with the 10-years option. Should these be the values to be used in the objective function (1)?

In reality these values should not be summed up together, because they represent values in different metrics. One easier way of solving this problem is resorting to the concept of Equivalent Annual Cost (EAC). The idea of the EAC is to consider a cost per period, such that if incurred each year during the whole planning period we would end up with the same PV associated with the cost of the facility itself. The EAC can be defined as follows, where a_r^T represents the annuity factor and *T* is the considered planning horizon [2]:

$$EAC = \frac{PV}{a_r^T} \qquad (7)$$

$$a_r^T = \frac{1-(1+r)^{-T}}{r} \qquad (8)$$

[2] The possibility of having two different facilities in operation in the same location can be easily incorporated into the SPLP by considering two potential fictitious locations that correspond to the same physical location.

Considering our example, we would end up with EAC' =22 265€ and EAC'' =33 398€. These values could be used as the facilities' fixed costs in SPLP.

If all facilities have exactly the same lifespan, then EAC or PV are two equivalent approaches.

2.2. Assignment costs

Let us now consider costs c_{ij}. These costs should represent the assignment costs: how much does it cost to assign client *j* to the facility located at *i*. In order to define these costs properly, we need to define the time period associated with these costs. Either c_{ij} could represent the costs incurred during one time period, or it could represent the cost of assigning client *j* to facility *i* during the whole planning horizon.

These assignment costs will be added to the total fixed costs, so care has to be taken to ensure that we are considering coherent metrics.

If we have defined the fixed costs as being equal to the PV of the costs flow during the planning horizon, then we should also consider the PV of the assignment costs during the planning horizon. If these costs are constant during the planning horizon then the PV can be easily calculated by (9):

$$PV = \frac{c_{ij}}{a_r^T} \tag{9}$$

If we have chosen to define the facilities' fixed costs as being equal to EAC, then we should only consider the assignment costs for one time period.

2.3. Example

As can easily be seen, whatever the choice made by the modeler, the optimal solution obtained will be dependent on two important model parameters: the planning horizon and the discount rate used. These parameters are not explicitly present in the model, but will have a determinant role in the optimal solution calculated.

Let us now illustrate these concepts with a simple example. Consider a problem with 5 potential locations where we can open facilities, and 10 clients that have to be assigned to an open facility. The spatial distribution of clients and potential locations for facilities is represented in figure 1. For each facility that is opened, we will incur in a fixed opening cost (that includes the operating cost during the first year), and a fixed annual operating cost. The value of each facility will depreciate at a rate of 20% per operating year, allowing us to estimate its salvage value at the end of the planning horizon. Assignment costs are constant throughout the planning horizon. Tables 2 and 3 depict this information.

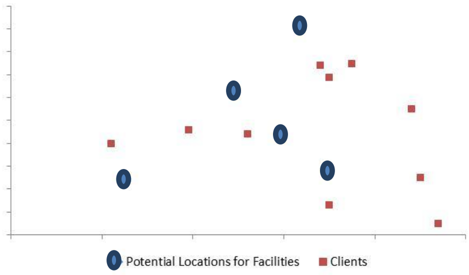

Figure 1 – Spatial distribution of clients and potential locations for facilities

Table 2 – Facilities' fixed and operating costs

Facility	Fixed opening cost	Fixed annual operating cost
1	5740	287
2	7493	187
3	4200	840
4	5586	559
5	1000	1000

Table 3 – Annual assignment costs

		Facilities				
		1	**2**	**3**	**4**	**5**
Clients	**1**	454	290	451	408	603
	2	346	182	178	300	495
	3	407	243	404	361	221
	4	469	327	288	445	600
	5	516	446	481	418	133
	6	473	462	486	549	149
	7	450	430	269	548	377
	8	348	328	167	446	479
	9	525	477	538	463	551
	10	289	453	302	571	333

Let us consider a planning horizon of 10 years, and a discount rate equal to 5%. The optimal solution to this problem would be to open facility 2 only, as depicted in figure 2.

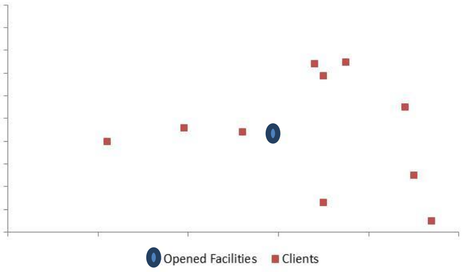

Figure 2 – Opened facility, for T=10 and r=5%

If we know consider a discount rate equal to 10%, then the optimal solution would be to open facility 3 instead (figure 3).

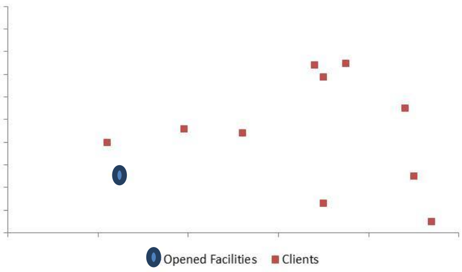

Figure 3 – Opened facility, for T=10 and r=10%

If we now consider a discount rate of 10%, but with a planning horizon of 5 years, then the optimal solution would be to open facility 5 only (figure 4).

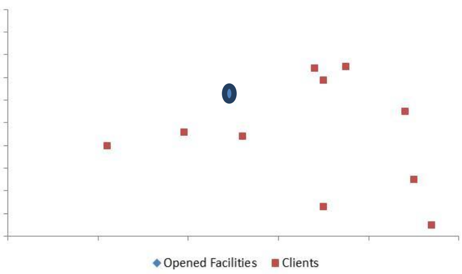

Figure 4 – Opened facility, for T=5 and r=10%

3. Computational results

To assess the influence that the definition of different planning horizons and discount rates could have on the optimal solution, several simple plant location instances were randomly generated and solved. The instances were generated according to the following procedure:

1. Random generation of (x, y) coordinates in the plane, according to a uniform distribution and considering a 500×500 square. These coordinates correspond to the location of clients and potential locations for facilities.
2. Random creation of arcs between the network nodes, considering a probability of 75%.
3. Creation of arcs (not created in step 2) between nodes such that the Euclidean distance from one another is less than 50, with probability of 80%.
4. Generation of costs associated with arcs: for the first period, the costs are randomly generated according to a uniform distribution, in the interval [100,1100]. For $t > 1$, the cost associated to the arc in period t is equal to the cost in $t - 1$ plus a changing factor randomly generated corresponding to a variation between −10% and +10%.
5. For each time period, calculation of the shortest path between each client and each facility, using the Floyd–Warshall algorithm.
6. For each facility *i* and period *t*, random generation of fixed and maintenance and operational costs. Facilities can be of one of two types: high setup costs and low maintenance and operational costs, or low setup costs and high maintenance and operational costs. In the first case, fixed costs are randomly generated in the interval [2000,10000]. In the latter, the interval considered is [500,3500]. Maintenance costs are calculated as a percentage of fixed costs, randomly generated using a uniform distribution in the interval [0%,10%] or [20%,75%] according to the type of facility.

Table 4 shows the dimension of the randomly generated problems. In total 1620 problems were generated considered all facilities of the same type, and another 1620 problems were generated considering facilities of different types (the choice of the type of facility was randomly generated with equal probabilities).

Table 4 – Dimension of randomly generated instances

Number of time periods	**Discount rate**	**Number of potential locations for facilities**	**Number of clients**
5	0%	10	25
10	5%	20	50
25	10%	50	100
		1000	200
			500
			1000

The aim of these computational results is the following: to see if the choice of the planning horizon and the discount rate does or does not influence the optimal solution, and how much could we lose if these two parameters were not appropriately chosen.

Two different types of experiments were carried out:

1. Considering the discount rate fixed, change the planning horizon: this will allow us to see how much we can lose if we consider a solution calculated with a given planning horizon, but then the facilities stay in operation during a different planning horizon.

2. Considering the planning horizon fixed, change the discount rate: this will allow us to see the influence of the discount rate.

As an example, consider that for a given problem a discount rate of 5% and a planning horizon of 5 years were considered. The optimal solution is calculated, but after implementing the solution it was decided that the facilities would be operating during 10 years. How much are we loosing because we did not consider a correct planning horizon right from the beginning?

Table 5 shows the results obtained when we consider the discount rate fixed and a planning horizon of 5 years when taking the decision. We then calculate the minimum, average and maximum loss in the objective function value if, in fact, the facilities stay in operation during 10 or 25 years. Similar results are presented in tables 6 and 7, for planning horizons of 10 and 25 years.

Tables 8 to 10 show similar results, but now when we consider solving the model with a given discount rate, and then change this discount rate.

Table 5 – Planning horizon equal to 5 years, discount rate fixed.

		T=10			T=25		
M	**N**	**Min**	**Average**	**Max**	**Min**	**Average**	**Max**
10	25	0.00%	0.49%	3.69%	0.00%	3.78%	15.96%
10	50	0.00%	0.45%	4.67%	0.00%	4.06%	13.57%
10	100	0.00%	0.64%	3.01%	0.00%	3.03%	8.81%
10	200	0.00%	0.43%	1.69%	0.00%	1.70%	4.92%
10	500	0.00%	0.26%	0.78%	0.00%	0.54%	1.21%
10	1000	0.00%	0.02%	0.29%	0.00%	0.03%	0.45%
20	25	0.00%	0.71%	7.53%	0.00%	5.89%	24.13%
20	50	0.00%	0.84%	4.39%	0.00%	3.18%	12.02%
20	100	0.00%	0.62%	2.66%	0.00%	3.13%	10.41%
20	200	0.00%	0.63%	2.03%	0.54%	3.31%	7.01%
20	500	0.00%	0.45%	1.19%	0.21%	2.45%	4.89%
20	1000	0.00%	0.22%	0.60%	0.00%	0.79%	1.87%
50	100	0.00%	0.72%	2.44%	0.00%	4.63%	12.62%
50	200	0.00%	0.67%	1.87%	0.16%	3.80%	11.24%
50	500	0.04%	0.58%	1.90%	0.70%	3.39%	8.82%
50	1000	0.01%	0.45%	0.95%	0.75%	2.48%	5.06%
100	200	0.00%	0.67%	1.85%	1.05%	4.87%	13.81%
100	500	0.01%	0.65%	2.04%	0.76%	3.81%	9.57%

Table 6 – Planning horizon equal to 10 years, discount rate fixed.

		T=5			T=25		
M	**N**	**Min**	**Average**	**Max**	**Min**	**Average**	**Max**
10	25	0.00%	0.38%	2.42%	0.00%	0.93%	5.72%
10	50	0.00%	0.28%	1.57%	0.00%	1.08%	3.90%
10	100	0.00%	0.56%	3.78%	0.00%	0.76%	2.72%
10	200	0.00%	0.31%	1.49%	0.00%	0.37%	1.83%
10	500	0.00%	0.21%	0.75%	0.00%	0.07%	0.41%
10	1000	0.00%	0.03%	0.35%	0.00%	0.00%	0.08%
20	25	0.00%	0.50%	3.37%	0.00%	1.91%	9.01%
20	50	0.00%	0.84%	3.95%	0.00%	0.73%	3.84%
20	100	0.00%	0.43%	1.62%	0.00%	0.87%	4.26%
20	200	0.00%	0.46%	1.22%	0.04%	0.81%	2.36%
20	500	0.00%	0.35%	1.16%	0.00%	0.63%	1.48%
20	1000	0.00%	0.19%	0.67%	0.00%	0.16%	0.59%
50	100	0.00%	0.56%	2.55%	0.00%	1.39%	4.45%
50	200	0.00%	0.56%	2.39%	0.00%	1.08%	4.27%
50	500	0.12%	0.52%	1.78%	0.00%	0.94%	3.03%
50	1000	0.06%	0.49%	0.87%	0.12%	0.64%	1.46%
100	200	0.00%	0.62%	1.95%	0.00%	1.46%	4.90%
100	500	0.04%	0.47%	1.24%	0.08%	1.04%	2.95%

Table 7 – Planning horizon equal to 25 years, discount rate fixed.

		T=5			T=25		
M	N	Min	Average	Max	Min	Average	Max
10	25	0.00%	2.39%	9.18%	0.00%	1.23%	7.82%
10	50	0.00%	2.12%	10.04%	0.00%	1.16%	5.46%
10	100	0.00%	2.44%	11.82%	0.00%	0.77%	3.47%
10	200	0.00%	1.45%	4.39%	0.00%	0.45%	2.01%
10	500	0.00%	0.73%	1.77%	0.00%	0.10%	0.59%
10	1000	0.00%	0.08%	0.73%	0.00%	0.01%	0.22%
20	25	0.00%	2.71%	9.89%	0.00%	1.09%	8.57%
20	50	0.00%	3.10%	10.53%	0.00%	0.74%	4.30%
20	100	0.00%	2.01%	5.88%	0.00%	0.62%	2.53%
20	200	0.37%	2.31%	5.01%	0.00%	0.87%	2.86%
20	500	0.28%	1.69%	4.04%	0.00%	0.63%	1.84%
20	1000	0.00%	0.76%	1.90%	0.00%	0.21%	0.52%
50	100	0.00%	2.67%	6.39%	0.00%	0.97%	4.01%
50	200	0.12%	2.46%	7.32%	0.00%	0.78%	2.60%
50	500	0.77%	2.26%	5.84%	0.00%	0.76%	2.30%
50	1000	0.30%	1.92%	3.58%	0.05%	0.60%	1.59%
100	200	0.43%	2.83%	7.56%	0.00%	1.03%	2.48%
100	500	0.43%	2.35%	5.59%	0.06%	0.88%	2.80%

Table 8 – Discount rate equal to 0%. Planning horizon fixed.

		5%			10%		
M	N	Min	Average	Max	Min	Average	Max
10	25	0.00%	0.66%	5.88%	0.00%	1.81%	9.18%
10	50	0.00%	0.39%	2.08%	0.00%	1.51%	10.04%
10	100	0.00%	0.30%	1.88%	0.00%	1.23%	5.14%
10	200	0.00%	0.22%	1.13%	0.00%	0.72%	3.67%
10	500	0.00%	0.06%	0.46%	0.00%	0.34%	0.94%
10	1000	0.00%	0.00%	0.00%	0.00%	0.03%	0.32%
20	25	0.00%	0.29%	1.73%	0.00%	2.01%	9.30%
20	50	0.00%	0.54%	2.63%	0.00%	1.28%	7.04%
20	100	0.00%	0.40%	1.85%	0.00%	1.12%	4.31%
20	200	0.00%	0.27%	0.95%	0.01%	1.40%	5.01%
20	500	0.00%	0.21%	1.21%	0.00%	0.85%	2.69%
20	1000	0.00%	0.07%	0.28%	0.00%	0.32%	0.80%
50	100	0.00%	0.65%	3.01%	0.00%	1.68%	6.39%
50	200	0.00%	0.53%	1.75%	0.00%	1.31%	3.49%
50	500	0.00%	0.36%	1.28%	0.13%	1.13%	3.07%
50	1000	0.05%	0.28%	0.61%	0.07%	0.92%	2.50%
100	200	0.00%	0.55%	1.91%	0.05%	1.52%	4.13%
100	500	0.00%	0.47%	1.82%	0.02%	1.19%	3.76%

Table 9 – Discount rate equal to 5%. Planning horizon fixed.

		0%			10%		
M	N	Min	Average	Max	Min	Average	Max
10	25	0.00%	0.45%	3.37%	0.00%	0.45%	3.41%
10	50	0.00%	0.40%	2.54%	0.00%	0.37%	2.51%
10	100	0.00%	0.42%	1.54%	0.00%	0.35%	1.41%
10	200	0.00%	0.17%	0.88%	0.00%	0.18%	1.15%
10	500	0.00%	0.05%	0.34%	0.00%	0.08%	0.44%
10	1000	0.00%	0.00%	0.00%	0.00%	0.01%	0.15%
20	25	0.00%	1.17%	7.56%	0.00%	0.50%	2.73%
20	50	0.00%	0.49%	2.90%	0.00%	0.25%	1.96%
20	100	0.00%	0.47%	2.44%	0.00%	0.24%	1.29%
20	200	0.00%	0.33%	0.75%	0.00%	0.36%	1.30%
20	500	0.00%	0.22%	0.58%	0.00%	0.20%	0.94%
20	1000	0.00%	0.04%	0.17%	0.00%	0.08%	0.27%
50	100	0.00%	0.76%	2.33%	0.00%	0.35%	1.42%
50	200	0.00%	0.58%	2.45%	0.00%	0.26%	1.09%
50	500	0.00%	0.48%	1.81%	0.00%	0.24%	0.69%
50	1000	0.02%	0.29%	1.12%	0.00%	0.21%	0.53%
100	200	0.00%	0.75%	2.99%	0.00%	0.30%	1.16%
100	500	0.00%	0.46%	1.50%	0.00%	0.25%	1.48%

Table 10 – Discount rate equal to 10%. Planning horizon fixed.

		0%			5%		
M	**N**	**Min**	**Average**	**Max**	**Min**	**Average**	**Max**
10	25	0.00%	1.81%	10.27%	0.00%	0.25%	2.80%
10	50	0.00%	1.58%	7.44%	0.00%	0.38%	3.33%
10	100	0.00%	1.37%	5.17%	0.00%	0.23%	1.97%
10	200	0.00%	0.70%	2.44%	0.00%	0.14%	0.90%
10	500	0.00%	0.27%	0.89%	0.00%	0.11%	0.41%
10	1000	0.00%	0.01%	0.12%	0.00%	0.01%	0.12%
20	25	0.00%	3.28%	19.50%	0.00%	0.66%	4.83%
20	50	0.00%	1.74%	8.19%	0.00%	0.23%	2.95%
20	100	0.00%	1.59%	5.96%	0.00%	0.24%	1.10%
20	200	0.08%	1.31%	3.32%	0.00%	0.40%	1.64%
20	500	0.00%	0.91%	2.76%	0.00%	0.25%	1.08%
20	1000	0.00%	0.25%	0.60%	0.00%	0.10%	0.42%
50	100	0.00%	2.52%	7.50%	0.00%	0.33%	2.14%
50	200	0.00%	1.97%	6.88%	0.00%	0.26%	0.88%
50	500	0.14%	1.59%	5.39%	0.00%	0.27%	1.14%
50	1000	0.18%	1.09%	3.27%	0.00%	0.22%	0.89%
100	200	0.05%	2.41%	8.23%	0.00%	0.37%	1.40%
100	500	0.01%	1.64%	4.95%	0.00%	0.26%	1.18%

From the computational results, we can conclude that when we consider similar facilities, about 30% of the problems' optimal solutions seem to be robust to changes in the planning horizon or discount rate. When we consider dissimilar facilities, this value decreases to 13%.

If all facilities have similar cost flow patterns, then the average loss is about 0.57%, and the maximum loss is equal 12.07%. If facilities have different cost flow patterns these numbers rise to 1.32% and 24.13% respectively.

It seems that the problem is more sensitive to changes in the planning horizon than changes in the discount rate. If we keep the number of potential locations fixed then, when the number of clients increases, the sensitivity regarding these parameters decreases. If we increase the number of potential locations for facilities, the sensitivity regarding these parameters increases.

4. Conclusions

Although the simple plant location problem does not consider explicitly time in its formulation, time dependent assumptions should always be explicitly defined, namely what are the planning horizon that is being considered and the value for the discount rate. This is particularly true if we are dealing with facilities that have different flow cost patterns, or different lifespans. And if we are dealing with real world applications this is crucial.

As a matter of fact, in every static location model that is used to represent a location problem, we should define time dependent assumptions. In dynamic location models, the planning horizon is explicitly defined as being part of the definition of the location variables, but in this case there is the need to explicitly determine the discount rate that is used and that enables us to sum up the costs incurred in different time periods.

Considering the influence that these parameters can have on the optimal solution, it can be considered a good practice to perform sensitivity analysis considering discount rate and planning horizon, to assess the sensitivity of each particular problem to time dependent assumptions.

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